Morphological modelling of rivers with erodible banks

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Abstract:
A bank erosion mechanism and provisions to account for the associated planform changes and input of bank erosion products are added to a two-dimensional, depth-averaged model of river morphology. The model is applied to a reach of the meandering gravel-bed River Ohrĕ (Eger) in the former state of Czechoslovakia. The agreement with observations is poor, but this can be ascribed to shortcomings in the flow and bed topography submodels rather than to shortcomings in the bank erosion submodel. Better results are expected when a three-dimensional flow model, equations for sediment mixtures and a bank accretion mechanism are included. This inclusion will have to be preceded by fundamental research on bank accretion mechanisms and on hiding and exposure effects in the relationship for the influence of gravity pull on sediment transport direction. © 1998 John Wiley & Sons, Ltd.

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INTRODUCTION
A common class of high resolution models of river morphology is two-dimensional in the horizontal plane. These models are based on depth-averaged, steady-flow equations, a volumetric sediment balance and formulae relating magnitude and direction of sediment transport to local flow field and bed topography. The first two-dimensional model of this kind was developed by van Bendegom (1947), but its application without the modern computer devices of the present was very laborious. Subsequent work on helical flow in river bends (e.g. Kalkwijk and de Vriend, 1980) and forces on sediment grains on a transversely sloping bed (e.g. Engelund, 1974; Odgaard, 1981) resulted in the model of Struiksma (1985), Struiksma et al. (1985) and Olesen (1987). Similar models were developed by Shimizu and Itakura (1985) and Nelson and Smith (1989). The earlier flow and bed topography model by Kennedy et al. (1983) does not fall into this class of models since it does not use the sediment balance. Instead, it uses an axisymmetrical relationship to link the bed topography directly to the local flow field. All models compute the evolution of river bed topography for a given planform with fixed banks. In natural rivers, however, planforms do change, through bank erosion and bank accretion.

This paper presents the extension of the two-dimensional morphological model of Struiksma, Olesen and co-workers with a bank erosion mechanism and provisions to account for the resulting planform changes and input of bank erosion products. The resulting model is applied to the River Ohrĕ in the former state of Czechoslovakia with two purposes. The first is to show the present predictive capabilities of the model. It is found that the agreement with observations is not good. Already, the reproduction of the bed topography is found to be poor, although basically the same model has given good results in applications to other rivers (Struiksma, 1985; Olesen, 1987). This leads to the second aim of the paper, to show how the construction and application of numerical models can guide further research. Better results

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are expected with a three-dimensional flow model, with equations for sediment mixtures and with a bank accretion model. Equations and solution techniques for three-dimensional flow are readily available, but fundamental research in a laboratory or in the field is needed for a proper formulation of the equations for sediment mixtures and the bank accretion model.

MATHEMATICAL MODEL

Coordinate system

The mathematical model is formulated in a horizontal, channel-fitted coordinate system \((s, n)\) which can be curvilinear and non-orthogonal. For clarity, however, the equations in this paper are written for a curvilinear, orthogonal coordinate system. Details on their non-orthogonal counterparts are given by Mosselman (1991, 1992). The \(s\) direction is taken to follow the streamwise direction of the river, i.e. more or less parallel to the streamlines, and boundary conforming at the banks. The \(n\) direction is the transverse direction. The coordinate increments are not dimensionless but represent metric distances. As a consequence, \(s\) and \(n\) are not independent, which implies that the order of cross-differentiation cannot simply be reversed but should be based on the relationship

\[
\frac{\partial^2 f}{\partial s \partial n} - \frac{\partial^2 f}{\partial n \partial s} = \frac{1}{R_s} \frac{\partial f}{\partial s} - \frac{1}{R_n} \frac{\partial f}{\partial n}
\]

in which \(f\) is a function of \(s\) and \(n\), and \(R_s\) and \(R_n\) are the radii of curvature of the \(s\) and \(n\) coordinate lines, respectively. The right-hand terms can vanish in a numerical implementation if the difference equations are properly discretized.

Flow model

The depth-averaged, steady-flow equations read as

\[
\begin{align*}
\frac{\partial u}{\partial s} + v \frac{\partial z_b}{\partial s} + g \frac{\partial h}{\partial s} + \frac{uv}{R_s} - \frac{v^2}{R_n} + \frac{g h^2}{h C^2} &= 0 \\
\frac{\partial v}{\partial s} + u \frac{\partial z_b}{\partial n} + g \frac{\partial h}{\partial n} + \frac{uv}{R_n} - \frac{u^2}{R_s} + \frac{g u v}{h C^2} &= 0 \\
\frac{\partial (hu)}{\partial s} + \frac{\partial (hv)}{\partial n} + \frac{hu}{R_n} + \frac{hv}{R_s} &= 0
\end{align*}
\]

in which \(u\) and \(v\) are the flow velocities in the \(s\) and \(n\) directions, respectively, \(z_b\) denotes bed level, \(h\) is the water depth, \(g\) is the acceleration due to gravity and \(C\) is the Chézy coefficient for hydraulic roughness. The streamwise coordinate lines and the streamlines of the flow are more or less parallel, but generally they will not match exactly. Hence, \(v\) cannot simply be neglected, and the streamline curvature is in general not equal to the curvature of the coordinate lines. By simplifying an expression for the streamline curvature derived by De Vriend (1976), Olesen (1982, 1987) obtains

\[
\frac{1}{R_f} = \frac{1}{R_s} - \frac{1}{u} \frac{\partial v}{\partial s}
\]

where \(1/R_f\) denotes the streamline curvature and \(1/R_s\) denotes the curvature of the streamwise coordinate line.
Curved streamlines induce secondary flow. The development of the secondary flow intensity is described by

$$\lambda_c \frac{\partial}{\partial s} \left( \frac{Ah}{R_*} \right) + \frac{Ah}{R_*} = \frac{Ah}{R_f}$$

(6)

in which $\lambda_c$ is the adaptation length of secondary flow, $A$ is a coefficient for the influence of secondary flow on the direction of the bed shear stress, depending on the hydraulic roughness and the eddy viscosity model applied, and $R_*$ is an 'effective' local radius of streamline curvature, which represents the intensity of the secondary flow. The secondary flow close to the bed adapts much faster to changing conditions than the higher parts, so that the value of $\lambda_c$ for the development of the bed shear stress owing to secondary flow is smaller than the one for the development of secondary flow as a whole.

Equation (6) has been taken from Olesen (1987) and differs slightly from the relationships used by Rozovskii (1957) and De Vriend (1981). The differences do not matter because all these relationships for secondary flow adaptation are approximations instead of exact expressions anyway.

The length scale of secondary flow adaptation is small compared with other length scales in the model. It is therefore omitted in the flow computation, thus assuming an immediate adaptation of secondary flow. The adaptation length is retained in the calculation of the direction of sediment transport (see section on the sediment model), because omission there would cause stability problems in the numerical model (Olesen, 1987).

The convective influence of the secondary flow deforms the horizontal velocity distribution of the main flow. Kalkwijk and De Vriend (1980) show that this effect can be represented in depth-averaged models by an additional acceleration or friction term. The streamwise friction term is then replaced as follows

$$\frac{gu^2}{hC^2} \rightarrow \frac{gu^2}{hC^2} + \frac{k_{sn} u^2}{h} \frac{\partial}{\partial n} \left( \frac{h^2}{R_*} \right)$$

(7)

in which $k_{sn}$ is the secondary flow convection factor. Formally, the equation thus obtained holds along characteristics that do not coincide with the streamlines but gradually shift towards the concave bank. The error in applying the equation along streamwise coordinate lines is negligible in mildly curved channels.

The depth-averaged flow model is not very accurate in regions close to steep banks, because lateral friction is neglected and the similarity hypothesis for vertical profiles used to obtain the depth-averaged equations is not valid here. An erroneous growth of the main velocity at concave banks is prevented by an artifice for the near-bank decay of the secondary flow intensity (Olesen, 1987).

**Bank erosion model**

Bank erosion can result from a variety of mechanisms in which many factors play a role. It can be a result of discharge-induced flow and sediment transport, but also of processes beyond the basic system of river morphology. Ikeda et al. (1981) present a model in which the bank erosion rate is proportional to the excess flow velocity above a certain critical value. They take this critical value to be equal to the reach-averaged flow velocity. Osman and Thorne (1988) model the retreat of cohesive banks as a sequence of erosion cycles in which toe erosion decreases bank stability by increasing the slope and the height of the bank, leading to the more dramatic erosion of the upper bank by mass failure. Bank stability characteristics are important for the modelling of mass failures, but can be ignored in the time-averaged description of the sequence, which Osman and Thorne (1988) term ‘parallel bank retreat’. The time-averaged behaviour can be described as a proportionality between bank erosion rate and excess shear stress, which, in a linear model, becomes equivalent to the excess flow velocity model of Ikeda et al. (1981).

During some time interval within the erosion cycle, higher and steeper banks are more likely to collapse than lower and less steep ones. The rate of bank retreat can hence be expected to be correlated with bank...
height and slope when looking at time-scales smaller than the period of an erosion cycle. For a uniform freeboard, that is a uniform difference between the water surface elevation and the top of the bank, correlations with bank height are equivalent to correlations with near-bank water depth. Models in which the bank erosion rate is proportional to excess near-bank water depth are proposed by Crosato (1989) and Odgaard (1989). Also, Hasegawa (1989) presents a relationship between near-bank water depth and bank erosion rate, but with opposite behaviour: larger near-bank water depths reduce the bank erosion rates. This is because Hasegawa’s relation is an integrated description with information on flow field and bed topography of the full cross-section, instead of an equation for an underlying mechanism (Mosselman and Crosato, 1991).

Both the excess shear stress mechanism and the excess bank height mechanism have been implemented in the numerical model (Mosselman, 1992, 1995). In this paper only the excess bank height mechanism is used.

\[
\frac{\partial n_B}{\partial t} = G \left( \frac{H - H_c}{H_c} \right) \quad \text{for} \quad H \geq H_c
\]

\[
\frac{\partial n_B}{\partial t} = 0 \quad \text{for} \quad H < H_c
\]

in which \( \frac{\partial n_B}{\partial t} \) denotes the rate of bank retreat, \( G \) is an erodibility coefficient, \( H \) is the total bank height and \( H_c \) is a critical bank height below which no bank erosion occurs. The total bank height is given by

\[
H = h_w + H_{fb}
\]

in which \( h_w \) is the near-bank water depth and \( H_{fb} \) is the freeboard. The mechanism thus formulated is equivalent to a mechanism in which bank retreat is a function of near-bank water depth when the freeboard is the same everywhere.

**Sediment model**

The magnitude of the sediment transport vector, \( |\mathbf{s}| \), is taken to be a function of the magnitude of the flow velocity vector, \( |\mathbf{u}| \), the water depth, \( h \), the Chézy coefficient, \( C \), the sediment grain size, \( D \), and the streamwise bed slope, \( \frac{\partial z_b}{\partial s} \), according to

\[
|\mathbf{s}| = \left( 1 - e \frac{\partial z_b}{\partial s} \right) f_s(|\mathbf{u}|, h, C, D)
\]

in which \( e \) is a coefficient for the effect of streamwise bed slopes on sediment transport, and \( f_s \) denotes the sediment transport rate predicted by a transport formula. The relationship implies that the sediment transport rate is determined by the local flow field and bed topography only, which is appropriate for bedload and situations in which the adaptation length of the suspended load is short compared with other length scales in the model. The sediment transport formulae of Meyer-Peter and Müller (1948) and Engelund and Hansen (1967) have been implemented.

The direction of the sediment transport vector is defined as the angle, \( \psi \), between the vector and the \( s \) direction. It is determined by the depth-averaged flow direction, the deviation of the near-bed flow direction from the depth-averaged flow direction owing to secondary flow and the effect of gravity on sediment grains on a sloping bed (Van Bendegom, 1947; Engelund, 1974). In the form used by Struiksma et al. (1985), \( \psi \) is given by

\[
\tan \psi = \frac{\nu}{u} - A \frac{h}{R_s} - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n}
\]
in which $f(\theta)$ is a function weighing the influence of gravity pull along a transverse bed slope. From experimental data presented by Zimmermann and Kennedy (1978), and using experience from the calibration of numerical models for river morphology, Struiksma and Crosato (1989) arrive at

$$f(\theta) = 0.85 \sqrt{\theta}$$  \hspace{1cm} (12)

in which $\theta$ denotes the Shields parameter.

Bank erosion products contribute to the sediment balance. Treating the input of bank erosion products as a boundary condition for the transverse sediment transport, $s_n$, at the banks gives problems in mathematical analyses because it is not compatible with the notion that the sediment transport is determined by the local flow field and bed topography. The bank erosion products are therefore incorporated as a source term in the continuity equation for sediment. A distribution function, $\delta(s, n)$, specifies how the supply of eroded bank material, $s_B(s)$, is distributed over the cross-section of the river. Then the full sediment balance reads

$$\frac{\partial z_i}{\partial t} + \frac{\partial s_i}{\partial x} + \frac{\partial s_n}{\partial n} + \frac{R_i}{R_n} \sum_{j=1}^{2} s_B(s) \delta_i(s, n)$$  \hspace{1cm} (13)

in which $i = 1$ denotes the right bank where $n = n_B$, and $i = 2$ denotes the left bank where $n = n_B$. The distribution function satisfies

$$\int_{n_B}^{n_B} \delta_i(s, n) \, dn = 1$$  \hspace{1cm} (14)

A distribution over a narrow zone along the eroding bank is obtained by defining $\delta_i(s, n)$ as a Dirac delta function

$$\delta_i(s, n) = \lim_{\Delta n \to 0} \frac{1}{\Delta n}$$  \hspace{1cm} for  \hspace{1cm} $n_B(s) < n < n_B(s) + \Delta n$

$$\delta_i(s, n) = 0$$  \hspace{1cm} for  \hspace{1cm} $n < n_B(s) \lor n > n_B(s) + \Delta n$$  \hspace{1cm} (15)

An analogous definition holds for $\delta_2(s, n)$. The corresponding difference equation used in the numerical model implies that bank erosion products are distributed evenly over the area of computational cells next to the banks. The input of bank erosion products, $s_{B_i}$, is equal to

$$s_{B_i} = (1 - \omega) H \left| \frac{\partial n_i}{\partial t} \right|$$  \hspace{1cm} (16)

in which $\omega$ is a washload factor.

**Computational procedure**

The computation of river planimetry can be decoupled from the computation of bed topography when bank retreat is much slower than degradation or aggradation of the bed. This is a reasonable assumption for cohesive banks. Furthermore, the flow is assumed to be quasi-steady, which means that the computation of bed topography can be decoupled from the flow computation as well. The equations are hence separated into a steady-flow model, a bed deformation model and a bank migration model. The corresponding computational procedure consists of three steps. In the first step, the flow field is computed while keeping the bed and bank configuration fixed. Sediment transport rates and bank migration rates are calculated from the flow field. In the second step, bed level changes are computed from the sediment transport gradients and the input of bank erosion products. Finally, bankline changes are calculated from the bank migration rates.
APPLICATION TO OHŘE RIVER, BOHEMIA, CZECHOSLOVAKIA

Introduction

The numerical model was applied to a reach of the meandering gravel-bed River Ohře (Eger) in the former state of Czechoslovakia. This river springs in the Fichtel Mountains in Germany and is a tributary to the River Labe (Elbe), see Figure 1. Its total length is about 300 km. The reach under consideration is about 15 km from the confluence, at the Ohře Meanders near Hostěnice, shown in Figure 2. The Pístý Woods are a nature reserve where the meander bends are allowed to migrate freely. The shape of the river in Figure 2 is interesting in view of a discussion on the extent to which the skewing of river bend planforms can be ascribed to geometric non-linearities or to lateral confinements. The coexistence here of sharp skewed bends on the left, where the river impinges on hills, and round bends on the right supports Ikeda’s (1989) opinion that the sharp skewed bends of the Beaver River in Alberta, Canada, shown by Parker et al. (1982, 1983), are primarily a result of the confinement within a narrow valley. Similar phenomena can be observed in the numerical simulations by Sun et al. (1996), where bends touching the meander belt borders are generally sharper and more skewed than bends completely inside the meander belt.

Schematization and data

The computational grid consisted of $25 \times 69$ points and is shown in Figure 3. Grid point numbers are indicated with $x$ in the streamwise and $Z$ in the transverse direction. Only two bends of the Ohře Meanders were modelled. The average longitudinal slope of the last 100 km of the River Ohře is 1.3 m/km. The reach under consideration at about 15 km from the confluence is less steep, however, which might be a result of an overall concave shape of the longitudinal profile as well as to sedimentation upstream of the Hostěnice weir. A longitudinal slope of 0.14 m/km was used in the computations. The average river width is 54 m in this reach.
Figure 2. The Ohrě Meanders near Hostěnice. Contour lines indicate heights in m above sea level.

Figure 3. Computational grid for River Ohrě with streamwise grid point numbers, \( \xi \).
A discharge duration curve was determined from a data file containing processed discharge data. Discharges below 55 m$^3$/s have been omitted because they did not produce any substantial sediment transport. Discharges above the bank-full discharge of 200 m$^3$/s have been reduced according to estimates of the distribution of discharges over the main channel and the floodplains. The duration curve with processed discharge data is shown in Figure 4, together with a schematization in which the discharge is 75 m$^3$/s during 90% of the time and 200 m$^3$/s during 10% of the time. The suitability of this schematization is discussed by Mosselman (1992).

Despite the quasi-steady flow assumption, modelling a varying discharge is still possible by using different discharges in different computational steps, as long as the discharges are kept constant within each of the steps. Here the discharges immediately preceding the bed topography survey were not known, so computations were made for the two discharges of the schematization, 75 m$^3$/s and 200 m$^3$/s, separately. The results were expected to be envelopes of the possible outcomes from computations with a varying discharge.

The Chezy coefficient for hydraulic roughness, $C$, varies from 65 to 70 m$^{1/2}$/s. A constant value of 67.5 m$^{1/2}$/s was chosen for the computations. Assuming logarithmic vertical velocity profiles, the corresponding coefficient for the influence of secondary flow on the direction of the bed shear stress, $A$, is 11 and the corresponding secondary flow convection factor, $k_{sn}$, is 0.4 (see Olesen, 1987). In sharp bends, however, $k_{sn}$ can be neglected in order to reduce the errors that arise from not reproducing secondary flow cells in the model (De Vriend and Geldof, 1983). Furthermore, Olesen (1987) finds that calibration in mildly curved bends sometimes requires values of $k_{sn}$ up to twice the theoretical value. He is not sure whether this is caused by a transverse variation of the alluvial roughness or by an underestimation of the influence of secondary flow convection, but A. M. Talmon (personal communication, 1991) concludes from curved flume experiments that it must be ascribed to the transverse roughness variation. The distribution of $k_{sn}$ displayed in Figure 5 was used in the computations, ensuring that $k_{sn} = 0$ in the sharp bend and $k_{sn} = 0.8$ in the mildly curved reach.

The locations of the banks had been mapped during two independent surveys, one in 1975 and 1976, and one in 1989. The banks did migrate in the period in between. A few bank material samples were taken during a visit in 1990. Nearly all samples were cohesive, but their composition varied considerably. Most samples consisted of a mixture of sand, clay, organic material and vegetation roots, but sometimes one or more of these components were absent. Bank erodibility parameters were not determined from the samples, but assigned uniform values by trial and error during calibration. The washload factor was chosen to be 1, because a mathematical analysis and trial computations indicated that the influence of bank erosion
products on two-dimensional bed topography patterns can be neglected in rivers with low banks and only minor width changes (Mosselman, 1992). The maximum sediment production from bank erosion per unit channel length in this river reach is of the order of $10^{-3}$ times the sediment transport per unit river width.

Detailed bank elevation data were not available, but it was assumed that the freeboard was more or less uniform in the reach. This assumption may be questionable in view of the suggestion in the previous section that the sharp skewed bends on the left side of the river are a result of the impingement on valley walls. The question is, however, whether the top of the bank should be defined as the pivot point of the outer bend cut in the valley wall or as the top of the hillslope. The pivot point might be a fair definition for relatively short time spans, whereas inclusion of the hillslope might be necessary when studying the long-term evolution of the river.

Because of the assumed uniform freeboard, the dependence of bank erosion on bank height could be formulated as a dependence on near-bank water depth. Expecting inaccuracies in the computed flow velocities and water depths close to the banks, it was assumed that it was better to correlate bank erosion with values computed further away from the banks, modifying $G$ and the critical near-bank water depth accordingly. Bank erosion was therefore related to water depths at $\eta = 5$ instead of $\eta = 1$ and at $\eta = 21$ instead of $\eta = 25$. Furthermore, bank erosion was assumed to take place during high discharge only.

Bed topography data stemmed from the survey in 1975 and 1976. The bed consists of sand and gravel with a wide distribution of grain sizes. Figure 6 shows a few grain size distribution curves. The grains were sorted in the transverse direction. Under a constant discharge, grain sorting is such that the sediment in the deeper pools is coarser than the sediment on the shallower bars (see Parker and Andrews, 1985), but here the opposite was observed. An explanation is that fine sediment brought into suspension during a flood had settled in the pools afterwards, whereas the smaller flow velocities at lower discharges could not remove the coarser sediment from the bars. Grain sorting was not taken into account, however. The computations were executed for uniformly distributed sediment with a median grain size, $D_{50}$, of 1.5 mm and a mean grain size, $D_m$, of 1.9 mm.

The sediment transport formulae of Engelund and Hansen (1967) and Ackers and White (1973) give the best results in one-dimensional morphological computations of a longer stretch of the river (E. Zeman, personal communication, 1990), but for the smaller flow velocities in the less steep reach considered here, the sediment transport formula of Meyer-Peter and Müller (1948) is more appropriate. The latter was used.

The unperturbed value of the function for the influence of gravity pull along a transverse slope, $f(\theta_0)$, was taken to be 0.22 for the discharge of 75 m$^3$/s and 0.30 for the discharge of 200 m$^3$/s.
Computational results and discussion

Computed planform evolutions are shown in Figures 7 and 8. Only one step of bank migration was computed, instead of a series of bank migration steps with intermediate equilibrium flow fields and bed topographies. For the larger bank erodibility in Figure 8, this computation with only one bank migration step led to incorrect overmigration, but provided a clear visualization of the initial rates of bank retreat. Both figures show only locally good agreement between computed and measured bank migration. It would have been easy to use locally different values for the bank erodibility parameters to obtain perfect agreement, but such a calibration would be misleading. It is more useful to identify the causes of the deviations and to formulate recommendations to improve the predictive capabilities of the model.

Apart from a lack of more data, three main causes for the deviations can be identified. First, bank erosion as a function of excess near-bank water depth with $G/H_c = 0.1 \times 10^{-6} \text{ s}^{-1}$ and $H_c - H_{fb} = 3.1 \text{ m}$
bank retreat might have been caused by, for instance, excess near-bank flow velocities, ice effects or overbank flow. Mosselman (1992) does not find better results when bank erosion is related to near-bank flow velocities, but B. Júza (personal communication, 1990) does find a correlation between the observed bank erosion rates and the computed cross-bank flow velocities in a two-dimensional depth-averaged simulation of the 1981 flood that severely inundated the village of Písty. Secondly, Figures 7 and 8 show that the banks advanced as well. The model does not contain any mechanism for bank accretion and, more generally, bank accretion has received much less attention in research than bank erosion. Thirdly, the poor agreement between the computed and the observed cross-sections shown in Figures 9 and 10 shows that the computation of flow field and bed topography in the river is inadequate, although previous computations of the bed topography of sand-bed rivers with basically the same model have been successful (Struiksma, 1985; Olesen, 1987). The present computations underestimated point bar heights and transverse bed slopes, and did not reproduce the reverse inclination of bed slopes near the concave bank. This third reason is the most important, because improved modelling of the mechanisms of bank advance and retreat does not make much sense if the variables that control these mechanisms are not computed correctly.

Figure 8. Measured and computed planform evolution over 14 years, the computation being based on only one step of bank migration. Bank erosion was taken to depend on excess near-bank water depth with \( G/H_c = 0.4 \times 10^{-6} \) s\(^{-1} \) and \( H_c - H_{fb} = 3.1 \) m

Figure 9. Computed water depths for \( Q = 75 \) and 200 m\(^3\)/s and measured water depths as a function of grid point number, \( \eta \), along the transverse coordinate line \( \xi = 40 \)
The deviations between the computed and the observed cross-sections in Figures 9 and 10 can be ascribed to the depth-averaged description of the flow field and neglect of grain sorting. The depth-averaged flow model yields inaccurate results in regions close to steep banks where the similarity hypothesis for vertical profiles of the velocity does not hold. Furthermore, the parameterization of secondary flow effects in Equations (7) and (11) does not reproduce the effects of the second, counter-rotating secondary flow cells that occur at sharp bends. It should also be noted that a different secondary circulation may occur when overbank flow crosses the main channel during a flood.

The reproduction of the observed grain sorting requires not only a morphological model for a mixture of different sediment size fractions, but also running this model with a varying discharge, because the observed coarser sediment on the bars and finer sediment in the pools is probably the result of bed evolution during a flood followed by a period of lower discharges. The high point bars might even be armoured remains of the bed formed during a large flood. The formulation of a two-dimensional morphological model for a mixture of different sediment size fractions would require experimental research on hiding and exposure effects in Equation (12) for the influence of gravity pull on the direction of sediment transport.

CONCLUSIONS

Bank erosion has been added to a depth-averaged mathematical model of river morphology. Application to a reach of the River Ohře (Eger) in the former state of Czechoslovakia yielded poor results, but an unequivocal conclusion that this was a result of the formulation of the bank erosion mechanism or the calibration of the bank erodibility parameters could not be drawn. Another reason for the poor results might be the absence of a bank accretion mechanism in the model. Bank accretion is a complex phenomenon in which point bar growth, discharge variations, channel incision, vegetation and washload play a role, but research on bank accretion has generally received much less attention than research on bank erosion. Therefore, more research on bank accretion is recommended.

More importantly, the reproduction of flow field and bed topography was also inadequate, despite successful application of basically the same model to sand-bed rivers. This can be ascribed to the depth-averaged description of the flow field and the neglect of grain sorting. The shortcomings of a depth-averaged description of the flow can be overcome when a three-dimensional flow model is used, although correct reproduction of two or more secondary flow cells in one cross-section will depend strongly on the turbulence.
model applied. Grain sorting can be dealt with by including equations for sediment mixtures and by applying the resulting model for a sequence of different discharges. The proper formulation of equations for sediment mixtures would require experimental research to examine hiding and exposure effects in the relationship for the influence of gravity pull on the direction of sediment transport.

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